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EVOLUTION OF THE LONG WAVE FINITE PERTURBATIONS DURING NONSTEADY COMBUSTION OF POWDER*

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The problem of nonsteady combustion of powder is considered within the framework of the modified Zel'dovich—Novozhilov model /1,2/. The development of the finite surface perturbations and the initial temperature profile and studied under the assumptions that the characteristic distance at which the perturbations change significantly is much greater than the characteristic dimension of relaxation of the stationary temperature profile. The method of multiple scales is used to derive the equations describing the evolution in time of the combustion surface and temperature field in the three-dimensional space. Only the long wave structure is considered, and solutions discontinuous in the given approximation are constructed. The internal structure of the discontinuities is not studied.

The model /1,2/ of nonsteady combustion of powder (condensed phase) is based on the assumption that the thermal relaxation of the condensed phase is the only inertial phase in the gas plus powder system. It is also assumed that the chemical reaction taking place in the powder at the gas-condensed phase interface is quasistationary and occupies a region significantly smaller than that determined by the characteristic dimension of the thermal relaxation.

An analogous problem arises in the study of the process when high power radiation acts on matter /3,4/. The problem of nonsteady combustion of powder where the burning surface is flat, was studied in /1,2/. Basically, this consisted of investigating the stability of the steady state modes under small perturbations /1,2/. A self-similar mode of powder combustion was obtained in /5/, and nonlinear transition combustion modes were studied by numerical methods in /6/. The present paper attempts an analytic solution of the problem of powder combustion in the three-dimensional formulation under the condition that the external conditions, the initial temperature distribution and initial form of the surface all change sufficiently slowly.

1. Basic equations.

Let us denote by t_1 the characteristic time of variation in the external condition, and by l_1 the characteristic dimension over which the initial temperature distribution, the form of the surface and the erosion flux, all vary. We further assume that the following relations hold:

$$O(t_0/t_1) = O(l_0/l_1) = \varepsilon, \quad 0 < \varepsilon \ll 1 \tag{1.1}$$

where l_0 is the characteristic dimension and t_0 the characteristic time of thermal relaxation. We assume that the region $X > S(Y, Z, t)$ corresponds to the space occupied by the powder where $S(Y, Z, t)$ is the burning surface. The problem is formulated mathematically as follows:

$$\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y} + \frac{\partial^2 \Theta}{\partial Z^2} \tag{1.2}$$

$$\Theta(X, Y, Z, 0) = \Theta^\circ(X, Y, Z), \; S(Y, Z, 0) = S^\circ(Y, Z)$$
$$\Theta(S(Y, Z, t), Y, Z, t) = \Phi(\varphi, p(t), Q(Y, Z, t))$$
$$\Theta(\infty, Y, Z, t) = 0, \quad \varphi = \left|\frac{\partial \Theta}{\partial \mathrm{N}}(S(Y, Z, t), Y, Z, t)\right|$$
$$V_n = (\partial S/\partial t)\,[1 + (\partial S/\partial Y)^2 + (\partial S/\partial Z)^2]^{-1/2} = F(\varphi, P, Q)$$

Here Θ is the dimensionless temperature of the powder, t is time, $R = (X, Y, Z)$ is the vector of

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space variables, $\Theta^{\circ}(\mathbf{R})$ is the initial temperature distribution, Φ and F are given functions of their arguments, φ is the temperature gradient normal to the surface, $\mathbf{N}$ denotes the normal to the surface, V_n is the normal rate of motion of the surface, $X - S(Y, Z, t) = 0$ is the equation describing the burning surface, $P(t)$ is the gas pressure and $Q(Y, Z, t)$ is the velocity of the erosion flux. Solving the problem (1.2) yields $\Theta(\mathbf{R}, t)$ and $S(Y, Z, t)$ for the given $\Theta^{\circ}(\mathbf{R})$, $S^{\circ}(Y, Z)$, Φ, F, P, Q. We note that the determination of the functions F and Φ which can, generally speaking, depend also on $\mathbf{R}$, t and ε explicitly, is a complicated problem. The problem is reduced to that of solving the steady state transport equation in the gas phase $(X - S(Y, Z, t)) < 0$ under the parametric dependence of the variables on time. Solution of the heat and mass transfer in the gaseous phase is made easier by taking into account the large values of the reaction activation energy which makes it possible to apply the method of matching asymptotic expansions. For the one-dimensional case (plane surface of combustion) the method of determining the functions F and Φ is thoroughly discussed in /1,2/. No solution is available so far for the two-dimensional and three-dimensional case.

Since here we concern ourselves with the case when ε is small, it is possible to use the quasi one-dimensional variant of F and Φ. Therefore, in what follows, we shall regard F and Φ as the principal terms of expansions of the corresponding multidimensional analogs into series in ε, for fixed $\mathbf{R}_\varepsilon$ and t_ε. The system (1.2) admits a one-dimensional steady state solution

$$S = Vt, \Theta(X, Y, Z, t) = A \exp[-V(X - Vt)] \tag{1.3}$$
$$\varphi = VA, \ P = P^{\circ}, \ Q = 0$$

$$V = F(AV, P^{\circ}, 0), \ A = \Phi(AV, P^{\circ}, 0) \tag{1.4}$$

The constants V and A are obtained from the solution of the algebraic equations (1.4). Taking into account the fact that the initial distributions vary smoothly, we pass to new independent variables

$$\tau = \varepsilon t, \ \mathbf{r} = \varepsilon\mathbf{R}, \ \mathbf{r} = (x, y, z)$$

Then the problem (1.2) assumes the form

$$\begin{aligned}
&\partial\Theta/\partial r = \varepsilon\Delta\Theta, \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2 \\
&\Theta(\mathbf{r}, 0) = A^{\circ}(\mathbf{r}) \exp(-\varepsilon^{-1} f^{\circ}(\mathbf{r})), \quad f^{\circ}(s, \mathbf{r}) = 0, \quad \mathbf{r}_2 = (y, z) \\
&\Theta(S(\mathbf{r}_2 r), \mathbf{r}_2, \tau) = \Phi(\varphi, \mathbf{r}_2, \tau), \quad \varphi = \varepsilon \left| \frac{\partial\Theta}{\partial n}(s(\mathbf{r}_2, \tau)\mathbf{r}_2, \tau) \right| \\
&\Theta(\infty, \mathbf{r}_2, \tau) = 0 \\
&\varepsilon(\partial s/\partial r)[1 + \varepsilon^2 |\nabla_2 s|^2]^{-1/2} = F(\varphi, \mathbf{r}_2, \tau) \\
&s(\mathbf{r}_2, \tau, \varepsilon) = S(\mathbf{R}_2, t), \ \nabla = \partial/\partial\mathbf{r}, \ \nabla_2 = \partial^2/\partial\mathbf{r}^2
\end{aligned} \tag{1.5}$$

We note that the steady state solution (1.3), (1.4) can be obtained from it by setting $A^{\circ}(\mathbf{r}) = A = \mathrm{const}$, $f^n(\mathbf{r}) = Vx$ and $A = O(1)$, $V = O(1)$. The method of regular perturbations is used in /7/ to obtain the solution of one-dimensional problem under the assumption that the initial distribution corresponds to a stationary distribution. The solution obtained however is not uniform with respect to the space variables.

2. Method of solution. We shall seek the solution of the problem (1.5) using the multiple scales method /8,9/. In the present case the method resembles the ray method of geometrical optics /9—12/ which is usually employed in solving linear problems. The method is used in /13/ to solve a nonlinear parabolic equation. Following /9/ we introduce the rapid variable

$$\eta = \xi(\mathbf{r}, t, \varepsilon)\varepsilon^{-1} = \varepsilon^{-1}[\xi_0(\mathbf{r}, \tau) + \varepsilon\xi_1(\mathbf{r}, \tau) + \varepsilon^2\xi_2(\mathbf{r}, \tau) + \ldots] \tag{2.1}$$
$$|\xi_i/\xi_{i-1}| = O(1)$$

Here $\xi_i(\mathbf{r}, \tau)$ are unknown functions which are determined in the course of solving the problem. We shall seek a solution of (1.5) in the form of a uniformly suitable expansion

$$\Theta = \Theta_0(\eta, \tau, \mathbf{r}) + \varepsilon\Theta_1(\eta, \mathbf{r}, \tau) + \varepsilon^2\Theta_2(\eta, \mathbf{r}, \tau) + \ldots, \tag{2.2}$$
$$|\Theta_i/\Theta_{i-1}| = O(1)$$
$$s(\mathbf{r}_2, \tau) = \varepsilon^{-1}[s_0(\mathbf{r}_2\tau) + \varepsilon s_1(\mathbf{r}_2, \tau) + \ldots] \tag{2.3}$$

Taking into account (2.1) and passing from $(\tau, \mathbf{r})$ to $(\tau, \mathbf{r}, \eta)$, we have

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial \tau} + \varepsilon^{-1}\frac{\partial \xi}{\partial \tau}\frac{\partial}{\partial \eta} \tag{2.4}$$
$$\Delta = \varepsilon^{-2}|\nabla\xi|^2\frac{\partial^2}{\partial\eta^2} + \varepsilon^{-1}\left(\Delta\xi\frac{\partial}{\partial\eta} + 2(\nabla\xi\nabla)\frac{\partial}{\partial\eta}\right) + \Delta$$

Substituting the expansion (2.2) into the first relation of (1.5) and taking (2.4) into account, we obtain

$$|\nabla\xi_0|^2\frac{\partial^2\Theta_0}{\partial\eta^2} - \frac{\partial\xi_0}{\partial\tau}\frac{\partial\Theta_0}{\partial\eta} = 0 \tag{2.5}$$

and solving (2.5) we obtain

$$\Theta_0(\eta, \mathbf{r}, \tau) = a(\mathbf{r}, \tau)\exp(g\cdot\eta) + b(\mathbf{r}, \tau),\ g(\mathbf{r}, \tau) = (\partial\xi_0/\partial\tau)\,|\nabla\xi_0|^{-2} \tag{2.6}$$

The functions $a(\mathbf{r}, \tau)$ and $b(\mathbf{r}, \tau)$ are obtained from the equations for the subsequent terms of the expansions (2.2). We note that we can put $\xi_1(\mathbf{r}, \tau) = 0$, without loss of generality. This follows from the form of (2.6). Further, carrying out the expansion in ε, we have

$$|\nabla\xi_0|^2\frac{\partial^2\Theta_1}{\partial\eta^2} - \frac{\partial\xi_0}{\partial\tau}\frac{\partial\Theta_1}{\partial\eta} = \frac{\partial\Theta_0}{\partial\tau} - \left(\Delta\xi_0\frac{\partial\Theta_0}{\eta} + 2\nabla\xi_0\nabla\frac{\partial\Theta_0}{\partial\eta}\right) \tag{2.7}$$

$$|\nabla\xi_0|^2\frac{\partial^2\Theta_2}{\partial\eta^2} - \frac{\partial\xi_0}{\partial\tau}\frac{\partial\Theta_2}{\partial\eta} = \frac{\partial\Theta_1}{\partial\tau} - \left(\Delta\xi_0\frac{\partial\Theta_1}{\partial\eta} + 2\nabla\xi_0\nabla\frac{\partial\Theta_1}{\partial\eta}\right) - 2(\nabla\xi_0\cdot\nabla\xi_2)\frac{\partial^2\Theta_0}{\partial\eta^2} - \Delta\Theta_0 \tag{2.8}$$

From (2.6) we see that the necessary condition for constructing the uniformly suitable expansion (2.2) is, that the solution $\Theta_j(\eta, \mathbf{r}, \tau)$ $(j = 1, 2)$ should contain no terms of the type η and $\eta^i e^{g\eta}$ $(i \geqslant 1)$. To remove such terms from the solution (2.7) it is necessary that the following relations hold:

$$\partial b/\partial\tau = 0,\quad b(\mathbf{r},\ 0) = 0,\quad b(\mathbf{r},\ \tau) = 0 \tag{2.9}$$

$$\partial g/\partial\tau - 2g(\nabla\xi_0\cdot\nabla)g = 0 \tag{2.10}$$

$$\partial a/\partial\tau - 2\nabla\xi_0\cdot\nabla(ag) - ag\Delta\xi_0 = 0 \tag{2.11}$$

Similarly, considering the right-hand side of (2.8) we can see that the term $\boldsymbol{\Delta\Theta_0}$ yields terms of the form $|\nabla g|^2\eta^2\exp(g\eta)$. It is therefore necessary to put $|\nabla g|^2 = 0$. This, together with (2.10), yields $g = \text{const}$. We choose without loss of generality $\text{const} = -1$, and this yields the following Hamilton–Jacobi equation for $\xi_0(\mathbf{r}, \tau)$:

$$\partial\xi_0/\partial\tau + |\nabla\xi_0|^2 = 0 \tag{2.12}$$

while for $a(\mathbf{r}, \tau)$ (2.11) yields

$$\partial a/\partial\tau + 2(\nabla\xi_0\cdot\nabla)a + a\Delta\xi_0 = 0 \tag{2.13}$$

Equations (2.12), (2.13) can be solved using the method of characteristics /14/. Let us introduce the function $\mathbf{p} = \nabla\xi_0$, $q = \partial\xi_0/\partial\tau$. Then (2.12) will assume the form $q = -|\mathbf{p}|^2$. The characteristic equations (ζ is a parameter describing the motion along the characteristic):

$$\frac{d\mathbf{r}}{d\zeta} = 2\mathbf{p},\quad \frac{d\mathbf{p}}{d\zeta} = 0,\quad \frac{d\tau}{d\zeta} = 1,\quad \frac{dq}{d\zeta} = 0 \tag{2.14}$$
$$d\xi_0/d\zeta = q + 2|\mathbf{p}|^2 = |\mathbf{p}|^2$$

Taking into account (2.14) we obtain

$$d/d\zeta = \partial/\partial\tau + 2\mathbf{p}\cdot\nabla$$

and equation (2.13) assumes the form

$$da/d\zeta + a\nabla\cdot\mathbf{p} = 0 \tag{2.15}$$

Integrating (2.14) and (2.15) we obtain

$$\mathbf{r} = \mathbf{r}_0 + 2\mathbf{p}_0\zeta,\ \mathbf{p} = \mathbf{p}_0,\ q = q_0,\quad \tau - \tau_0 = \zeta;\quad \xi_0 - \xi_0{}^\circ = |\mathbf{p}_0|^2\zeta,\quad a = a_0\exp\left(-\int_0^\zeta\nabla\cdot\mathbf{p}\,d\zeta'\right)$$

where $\mathbf{r}_0, \mathbf{p}_0, q_0, \tau_0, \xi_0, a_0$ are the values of the corresponding quantities at $\zeta = 0$.

We formulate for $\xi_0(\mathbf{r},\ \tau)$ and $a(\mathbf{r},\ \tau)$ the following problem with boundary conditions:

$$\xi_0(x = s_0(\mathbf{r}_2,\ \tau)) = 0,\quad a(x = s_0(\mathbf{r}_2,\ \tau),\ \mathbf{r}_2,\ \tau) = \Phi,\quad \xi_0(\mathbf{r},\ 0) = f^\circ(\mathbf{r}),\ a(\mathbf{r},\ 0) = A^\circ(\mathbf{r}) \qquad (2.16)$$

The solution of (2.12) can be, generally speaking, discontinuous. This is apparent even from the fact that is we introduce $u(\mathbf{r},\tau) = \nabla\xi_0$ and take the gradient of (2.12), then we have

$$\partial u/\partial r + 2u\cdot\nabla u = 0$$

which represents an equation, well known in gas dynamics /15/, leading to the discontinuous solutions. When the initial and boundary conditions (2.19) are matched, i.e. when

$$f^\circ(s_0(\mathbf{r}_2, 0),\ \mathbf{r}_2, 0) = 0,\ A^\circ(s_0(\mathbf{r}_2, 0), 0) = \Phi$$

the solution of (2.12), (2.13) can have derivatives with discontinuities.

The appearance of discontinuities suggests that narrow zones exist in the $(\mathbf{r},\tau)$ space in the asymptotic solution (2.2), in which the approach adopted here cannot be used. Such discontinuities resemble shock waves of gas dynamics /15/. In order to study their structure, we must begin from the complete equations (1.5) and construct the corresponding new expansions. The equation describing the evolution of the combustion surface

$$\partial s_0/\partial r\,[1 + |\nabla_2 s_0|^2]^{-1/2} = F(\varphi_0,\ \tau,\ \mathbf{r}_2)\, s_0(\mathbf{r}_2,\ 0) = s_0^\circ(\mathbf{r}_2) \qquad (2.17)$$

will be dealt with below. We note that the quantity φ_0 is equal to

$$\varepsilon\left|\frac{\partial}{\partial x}\left(a(\mathbf{r},\tau)\exp(-\eta)\right|_{x=s^\circ(\mathbf{r}_2,\ \tau)} = \left|\frac{\partial\xi_0}{\partial x}\, a\right|_{x=s^\circ(\mathbf{r}_2,\ \tau)} + o(1)$$

divided by the cosine of the angle between the normal and the x-axis.

To find the solution of (2.2) in the neighborhood of $s^\circ(\mathbf{r}_2,\tau)$, we expand $\xi_0(\mathbf{r},\tau)$ into a series in $(x - s^\circ(\mathbf{r}_2,\tau))$:

$$\xi_0(\mathbf{r},\tau) = \alpha(\tau_2,\ \tau)(x - s^\circ) + o((x - s^\circ)) \qquad (2.18)$$

Substituting (2.18) into (2.12) and setting $x \to s^\circ$ we obtain

$$\alpha(\mathbf{r}_2,\tau) = \frac{\partial s_0}{\partial\tau}[1 + |\nabla_2 s_0|^2]^{-1} \qquad (2.19)$$

Calculating the projection on the normal to the surface yields

$$\varphi_0(\mathbf{r}_2,\tau) = a(s^\circ, \mathbf{r}_2, \tau)\frac{\partial s_0}{\partial\tau}[1 + |\nabla_2 s_0|^2]^{-1/2} \qquad (2.20)$$

We note that the temperature gradient is governed at every point of the surface by the same relation as in the steady state solution (1.3). Indeed we have

$$\varphi_0(\mathbf{r},\tau) = a(s^\circ,\ \mathbf{r}_2,\ \tau)\, V_n^\circ(s_0,\ \mathbf{r}_2,\ \tau) \qquad (2.21)$$

Thus by solving the algebraic equations entering (1.5) for a and $V_n^\circ = (\partial s_0/\partial\tau)[1 + |\nabla_2 s_0|^2]^{-1/2}$, we obtain two equations for determining the form and temperature of the surface. The absence of real roots of these equations indicates the absence of combustion-extinction. Thus we have the following expressions for determining $a(\mathbf{r}_2,\tau)$ and $s_0(\mathbf{r}_2,\tau)$:

$$\frac{\partial s_0}{\partial\tau}[1 + |\nabla_2 s_0|^2]^{-1/2} = \beta(\mathbf{r}_2,\tau) > 0,\quad s_0(\mathbf{r}_2, 0) = s_0^\circ(\mathbf{r}_2) \qquad (2.22)$$

$$a(\mathbf{r}_2,\ \tau) = \gamma(s_0(\mathbf{r}_2,\ \tau),\ \mathbf{r}_2\tau) > 0 \qquad (2.23)$$

where β and γ are known functions of their arguments. The Cauchy problem (2.22) can be solved using the method of characteristics

$$d\mathbf{r}_2/d\tau = -\beta(\mathbf{r}_2,\ \tau)\cdot\mathbf{p}_2(1 + |\mathbf{p}_2|^2)^{-1/2},\qquad \frac{ds_0}{d\tau} = \beta(\mathbf{r}_2,\tau)(1 + |\mathbf{p}_2|^2)^{-1/2},\quad \frac{d\mathbf{p}_2}{d\tau} = \nabla_2\beta,\quad \mathbf{p}_2 = \nabla s_0(r_2,\tau) \qquad (2.24)$$

and in the case when $\beta(\mathbf{r}_2,\ \tau) = \beta(\tau)$, the characteristic equation (2.24) yields (η is a parameter)

$$\mathbf{r}_2 - \eta = G(\eta)\,T,\ \ s_0 - s_0^\circ(\eta) = B(\eta)\,T \qquad (2.25)$$

$$T = \int_0^\tau \beta(s)\,ds,\quad G(\eta) = -\nabla_2 s_0^\circ|(1 + |\nabla_2 s_0^\circ|^2)^{-1/2}|_{r_2=\eta},\qquad B(\eta) = (1 + |\nabla s_0^\circ|^2)^{-1/2}|_{r_2=\eta}$$

3. Plane surface of combustion. Consider a particular case

$$s_0(\mathbf{r}_2, 0) = 0;\ a(\mathbf{r}) = a(x) \quad \text{and} \quad f^\circ(\mathbf{r}) = f^\circ(x),\ \Phi(\varphi, \tau, r_2) = \Phi(\varphi, \tau),\ F = F(\varphi, \tau)$$

Since the boundary and initial conditions are arbitrary, the solution of (2.12) and (2.13) and equations describing the surface

$$ds_0/d\tau = F(\varphi, \tau),\ s_0(0) = 0 \tag{3.1}$$

are all independent of the coordinates y and z. Two families of characteristics exist. One family originates on the axis $\tau = 0\ (0 \leqslant x \leqslant \infty)$ and the other on $x = s_0(\tau)$. For the first family $\tau_0 = 0$, $\zeta = \tau$ we have from (2.5)

$$x - x_0 = 2\frac{df^\circ}{dx}(x^\circ)\tau, \quad \xi_0 - f^\circ(x_0) = \left(\frac{df^\circ}{dx}(x_0)\right)^2 \tau \tag{3.2}$$

$$a(x, \tau) = A^\circ(x_0) \exp\left[-\int_0^\tau \frac{d^2 f}{dx^2}(x_0(\tau'))\right] \times \left[1 + 2\frac{d^2 f_0}{dx^2}(x_0(\tau'))\tau'\right]^{-1} d\tau'$$

and the other family has the form (γ is a parameter)

$$x = s_0(\gamma) + 2\frac{ds_0}{d\tau}(\gamma)(\tau - \gamma) \tag{3.3}$$

$$\xi_0(x, \tau) = \left[\frac{ds_0}{d\tau}(\gamma)\right]^2 (\tau - \gamma)$$

The value of the parameter $\gamma = \tau$ corresponds to the surface $x = s_0(\tau)$. In the case when the initial distribution is stationary, (3.2) yields for $A^\circ = 1$, $f^\circ(x) = x$:

$$x_0 = x - 2\tau,\ \xi_0(x, \tau) = x - \tau$$

$$a(x, \tau) = 1 \quad \text{when} \quad x \geqslant 2\tau$$

4. Two-dimensional surface of combustion. Let us consider the combustion of powder in the framework of the model given in /1,2/. Let the surface of combustion have the form $x = s_0^\circ(y, 0) = \omega(y)$ at the initial instant. Let also the erosion flux be absent. Then the equation describing the evolution of the surface has the form

$$\frac{\partial s_0}{\partial \tau} = V(\tau)\sqrt{1 + (\partial s_0/\partial y)^2}, \quad s_0(y, 0) = \omega(y) \tag{4.1}$$

where $V(\tau)$ is the local normal velocity of motion of the surface. It is convenient to introduce at this stage a new, time-like variable

$$T = \int_0^\tau V(\tau')\, d\tau'$$

Then from (4.1) follows

$$\partial s_0/\partial T = \sqrt{1 + (\partial s_0/\partial y)^2}, \quad s_0(y, T = 0) = \omega(y) \tag{4.2}$$

From (2.25) we have the following expressions for (4.2) (η is a parameter):

$$y - \eta = -\frac{d\omega}{d\eta}\left[1 + \left(\frac{d\omega}{d\eta}\right)^2\right]^{-1/2} T \tag{4.3}$$

$$s_0 - \omega(\eta) = \left[1 + \left(\frac{d\omega}{d\eta}\right)^2\right]^{-1/2} T$$

Consider the case when $\omega(y)$ is a periodic function. Then $d\omega/d\eta = 0$ when $\eta = \eta_n$ where η_n is a root of the equation $d\omega/d\eta\,(\eta = \eta_n) = 0$. We have

$$s_0 = T + \omega(\eta_n), \quad y = \eta_n$$

Let us assume, for the sake of definitness, that

$$\omega(y) = \delta \cos y,\ \delta > 0,\ \eta_n = n\pi,\ n = 0, 1, 2, \ldots$$

Then $\eta_n = n\pi$, $n = 0, 1, 2, \ldots$. From the first equation we obtain

$$(y - \eta)/T = \delta \sin\eta\, [1 + \delta \sin^2\eta]^{-1/2} = G(\eta) \tag{4.4}$$

Since $\omega(y)$ is periodic, it is sufficient to consider the case $-\pi \leqslant y \leqslant \pi$. We shall solve the equation (4.4) graphically. Fig.1 depicts the graph of the right-hand side of (4.4). We

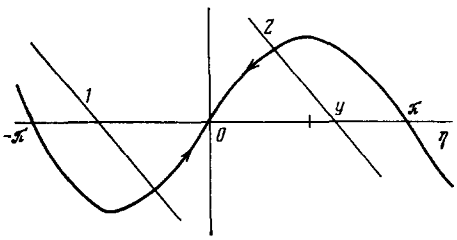

Fig.1

project for $-\pi < y < 0$ the straight line $1\ (y-\eta)\,T^{-1}$. The point of intersection of the straight line 1 with $G(\eta)$ tends, with increasing T, to the point $\eta = 0$, and as $T \to \infty$, (4.4) and the second equation of (4.3) together yield

$$\eta = \frac{y}{T} + o(T^{-1}), \quad -\pi < y < 0 \qquad (4.5)$$
$$s_0 = T + \delta + \frac{y^2}{2T} + o(T^{-1}) \quad \text{as} \quad T \to \infty$$

The first equation of (4.4) yields. at the points $\eta = -\pi$, $\eta = 0$ and $\eta = \pi$,

$$s_0 = \begin{cases} -\delta + T, & y = -\pi \\ +\delta + T, & y = 0 \\ -\delta + T, & y = \pi \end{cases} \qquad (4.6)$$

When $0 < y < \pi$, then the point of intersection of $G(\eta)$ with the straight line tends to $\eta = 0$ as $T \to \infty$. Thus we have

$$\eta = y/(\delta T) + o(T^{-1}),\ 0 < y < \pi \qquad (4.7)$$
$$s_0 = \delta + T + \frac{y^2}{2T} + o(T^{-1})$$

A similar construction can be carried out for other values of y. When $T \to \infty$, the form of the surface assumes, in accordance with (4.5)—(4.7), the form depicted in Fig.2 ($s_0 - T$). Thus in the scale ε^{-1} the surface of combustion tends to a plane except at a finite number of points such as in the case $\cos(y) = -1;\ y = (2k+1)\pi;\ k = 0,\ \pm 1,\ \pm 2$.

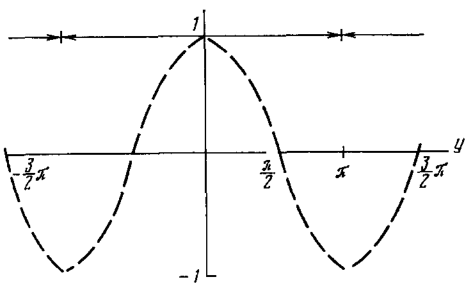

Fig.2

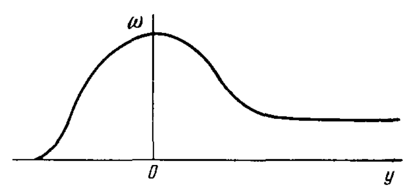

Fig.3

Generally speaking, when the surface profile is "twisted" near the points with large curvature, the proposed asymptotic method becomes inapplicable and the solution constructed here must be regarded as the outer /8,9/ solution. The structure of "projections" appearing on the surface of combustion at the points must be investigated by passing to the spatial variables R. We note that the surface of combustion can be constructed using the method analogous to the Huygens principle /16/ used in a number of works to describe the propagation of a gas flame, e.g. in /17,18/.

Next we consider the case $d\omega/dy\,(\pm\infty) = 0$. We can put $\omega(-\infty) = 0$ without loss of generality. We introduce the function $u(y, T)$:

$$s_0(y, T) = T - \int_{-\infty}^{y} u(\zeta, T)\,d\zeta$$

and in place of (4.2) we have

$$\frac{\partial u}{\partial T} + \frac{\partial}{\partial y}\sqrt{1+u^2} = 0, \quad u(y, 0) = -\frac{d\omega}{dy} \qquad (4.8)$$

Let now $\omega(y)$ be a monotonously decreasing and sufficiently smooth function $\omega(+\infty) = \omega_+ < 0$ with a nonzero derivative on the finite interval $(0, L)$. When $|d\omega/dy| < 1$, the radical in (4.8) can be expanded into a series and we can take into account only two terms of this expansion in the first approximation. Then as $T\to\infty$, the solution can be written, in the following explicit form /19/:

$$s_0(y,T)=\begin{cases}T, & y<0\\ T-y^2/(2T), & 0<y<\sqrt{\alpha T}\\ T+\omega_+, & \sqrt{\alpha T}<y\end{cases}$$
$$\alpha=-2\omega_+$$

We note that the expansion of the radical is allowed when $\varepsilon\ll|d\omega/dy|$, otherwise these terms will appear only in the subsequent orders of the expansion $s(x,y,t;a)$. Using above formulas we obtain the equation of the surface of combustion as $T\to\infty$ in the form $x-s_0(y,T)=0$. If on the other hand $\omega(y)$ is sifficiently smooth and has the form shown in Fig.3 ($\omega(0)=\max\omega(y)$) and $d\omega/dy\not\equiv 0$, of a finite segment, then using the approximation in which the above formulas was derived, we obtain for $T\to\infty$

$$s_0(y,T)=\begin{cases}T, & y<-\sqrt{\beta T}\\ T=y^2/(2T)+\beta/2, & -\sqrt{\beta T}<y<\sqrt{\alpha T}\\ T-(\alpha-\beta)/2, & \sqrt{\alpha T}<y\end{cases}$$
$$\beta=2\omega(0),\quad \alpha=2(\omega(0)-\omega_+),\quad \omega_+\gtrless 0$$

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